EFFECT OF MAGNETIC FIELD ON THE STRANGE STAR

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Abstract

We study the effect of a magnetic field on the strage quark matter and apply to strange star. We found that the strange star becomes more compact in presence of strong magnetic field.

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There are several different scenarios for estimation of the magnetic field strength at the surface of neutron star. These are: theoretical models of pulsar emission [1] the accretion flow in the binary X-ray sources [2] and observation of features in the spectra of pulsating X-ray sources which have been interpreted as cyclotron lines [3]. In a sample of more than 300 pulsars the range of values of the surface magnetic field strength runs into the interval $10.36 \leq \log(B)$ (G) ≤ 13.33 [4].

Very recently, several authors [5] have proposed two different physical mechanism leading to an amplification of some initial magnetic field in a collapsing star. Fields as strong as $B \sim 10^{14} \quad \sim 10^{16}$G , or even more, might be generated in new born neutron stars. In the interior of neutron star, it probably reaches $\sim 10^{18}$G. Therefore, it is advisable to study the effect of strong magnetic field on compact neutron stars.

There are strong reason for believing that the hadrons are composed of quarks , and the idea of quark stars has already been existed for about twenty years. If the neutron matter density at the core of neutron stars exceeds a few times normal nuclear density ($3n_0, n_0 = 0.15\ fm^{-3}$) a deconfining phase transition to quark matter may take place. As a consequence, a normal neutron star will be converted to a hybrid star with an infinite cluster of quark matter core and a crust of neutron matter. In 1984, Witten suggested that strange matter, e.g., quark matter with strangeness per baryon of order unity, may be the true ground state [6]. The properties of strange matter at zero pressure and zero temperature were subsequently examined, and it was found that strange matter can indeed be stable for a wide range of parameters in the strong interaction calculations [7]. Therefore, at the core, the strange quarks will be produced through the weak decays of light quarks (u and d quarks) and ultimately a chemical equilibrium will be established among the participants. Since, the strange matter is energitically favourable over neutron matter, there is a possibility that whole star may be converted to a strange star.

For a constant magnetic field along the z-axis ($\vec{A} = 0, \vec{H} = H(z) = H = constant$), the single energy eigenvalue is given by [8]

$$\varepsilon_{p,n,s} = \sqrt{{p_i}^2 + {m_i}^2 + q_i H(2n + s + 1)} \tag{1}$$

where n=0, 1, 2, ..., being the principal quantum numbers for allowed Landau levels, $s = \pm 1$ refers to spin up(+) and down(-) and p_i is the component of particle(species i) momentum along the field direction. Setting $2n + s + 1 = 2\nu$, where $\nu = 0, 1, 2...$, we can rewrite the single particle energy eigenvalue in the following form

$$\varepsilon_i = \sqrt{{p_i}^2 + {m_i}^2 + 2\nu q_i H} \tag{2}$$

Now, it is very easy to see that $\nu = 0$ state is singly degenerate, whereas, all other states with $\nu \neq 0$ are doubly degenerate. Then the thermodynamic potential in presence of strong magnetic field $H (> H^{(c)}$, critical value discussed later) is given by

$$\Omega_i = -\frac{g_i q_i H T}{4\pi^2} \int d\varepsilon_i \sum_\nu \frac{dp_i}{d\varepsilon_i} \ln[1 + exp(\mu_i - \varepsilon_i)/T]. \tag{3}$$

Integrating by parts and substituting

$$p_i = \pm\sqrt{\varepsilon_i^2 - m_i^2 - 2\nu q_i H}, \tag{4}$$

for all T, one finds

$$\Omega_i = -\frac{g_i q_i H}{4\pi^2} \int d\varepsilon_i \sum_\nu \frac{2\sqrt{\varepsilon_i^2 - m_i^2 - 2\nu q_i H}}{[exp(\mu_i - \varepsilon_i)/T + 1]} \tag{5}$$

where the sum over ν is restricted by the condition $\varepsilon > \sqrt{m^2 + 2\nu q H}$ and the factor 2 takes into account the freedom of taking either sign in eq(4). For T =0, therefore, approximate the Fermi distribution by a step function and interchange the order of the summation over ν and integration over ε,

$$\begin{aligned}\Omega_i &= -\frac{2 g_i q_i H}{4\pi^2} \sum_\nu \int_{\sqrt{m_i^2 + 2\nu q_i H}}^{\mu} d\varepsilon_i \sqrt{\varepsilon_i^2 - m_i^2 - 2\nu q_i H} \\ &= -\frac{2 g_i q_i H}{8\pi^2} \sum_\nu (\mu_i \sqrt{\mu_i^2 - m_i^2 - 2\nu q_i H} \\ &\quad -(m_i^2 + 2\nu q_i H) \ln[\frac{\mu_i + \sqrt{\mu_i^2 - m_i^2 - 2\nu q_i H}}{\sqrt{{m_i}^2 + 2\nu q_i H}}])\end{aligned} \tag{6}$$

Since the temperature $T << \mu$ at the core of neutron star, the presence of anti-particles can be ignored. Now instead of infinity the upper limit of ν sum can be obtained from the following relation

$$p_{Fi}{}^2 = \mu_i{}^2 - m_i{}^2 - 2\nu q_i H \geq 0, \tag{7}$$

where p_{Fi} is the Fermi momentum of the species i, which gives

$$\nu \leq \frac{\mu_i{}^2 - m_i{}^2}{2q_i H} = \nu_{max}{}^{(i)} \quad (nearest \;\; integer). \tag{8}$$

Therefore, the upper limit is not necessarily same for all the components. As is well known, the energy of a charged particle changes significantly in the quantum limit if the magnetic field strength is equal to or greater than some critical value $H^{(c)} = m_i{}^2 c^3/(q_i \hbar)$ (in G), where m_i and q_i are respectively the mass and the absolute value of charge of particle i, $\hbar$ and c are the reduced Planck constant and velocity of light respectively, both of which along with Boltzman constant k_B are taken to be unity in our choice of units. For an electron of mass 0.5 MeV, this critical field as mentioned above is $\sim 4.4 \times 10^{13}$G, whereas for a light quark of current mass 5 MeV, this particular value becomes $\sim 4.4 \times 10^{15}$G, on the other hand for s-quark of current mass 150 MeV, it is $\sim 10^{19}$G, which is too high to realise at the core of neutron star. Therefore, the quantum mechanical effect of neutron star magnetic field on s-quark has been ignored [9]. But in the present work, we have considered the possibility of having with and without the effect on s-quark by the presence of magnetic field. If the motion of s-quarks are not effected by the presence of strong magnetic field, the thermodynamic potential for this component for $T = 0$ is given by [7,10]

$$\Omega_s = -\frac{1}{4\pi^2}[\mu_s \sqrt{\mu_s{}^2 - m_s{}^2}(\mu_s{}^2 - 2.5 m_s) + 1.5 m_s{}^4 \ln(\frac{\mu_s + \sqrt{\mu_s{}^2 - m_s{}^2}}{m_s})]. \tag{9}$$

In our study, we assume that strange quark matter is charge neutral and also chemical equilibrium, then

$$\mu_d = \mu_s = \mu = \mu_u + \mu_e, \tag{10}$$

and charge neutrality conditions gives

$$2n_u - n_d - n_s - 3n_e = 0. \tag{11}$$

The baryon number density of the system is given by

$$n_B = \frac{1}{3}(n_u + n_d + n_s). \tag{12}$$

Using above eqs(10, 11, 12) one can solve numerically for the chemical potentials of all the flavours and electron. Now, for $T = 0$, we have the number density of the species i $(u,\ d,\ s,\ e)$

$$n_i = \frac{g_i q_i H}{4\pi^2} \sum_\nu \sqrt{{\mu_i}^2 - {m_i}^2 - 2\nu q_i H}. \tag{13}$$

The number density of s-quark in absence of magnetic field is given by

$$n_s = \frac{({\mu_s}^2 - {m_s}^2)^{3/2}}{\pi^2}. \tag{14}$$

In fig. (1), we have plotted strangeness fraction (n_s/n_B) as function of baryon density. Curve (a) is for low magnetic field($H = 10^{14}$ G) and curve (b) is for with high magnetic field($H = 10^{18}$ G). Whereas, the curve (c) is for with high magnetic field($H = 10^{18}$ G) without the presence of magnetic field in s-quark. In curve (b) and (c), the strangeness fraction is showing an oscillating behaviour as consecutive Landau levels are passing the Fermi level. However, in curve (a), there is no oscillating behaviour because of low magnetic field.

The total energy density and the external pressure of the strange quark matter is given respectively by

$$\begin{aligned} \varepsilon &= \sum_i \Omega_i + B + \sum_i n_i \mu_i \\ p &= -\sum_i \Omega_i - B, \end{aligned} \tag{15}$$

where $i =\ u,\ d,\ s,\ e$. Here, we have considered the conventional bag model for the sake of simplicity in presence of magnetic field. We are assuming that quarks are moving freely (non-interacting) within the system and as usual the current masses of both u and d quarks are extremely small, e.g., 5 MeV each, whereas, for s-quark the current quark mass is to

be taken 150 MeV. We choose the bag pressure B to be 56 $MeV\ fm^{-3}$. Also, we set the magnetic field to be $H = 10^{14}$ G for low and $H = 10^{18}$ G for high fields in our calculations.

We have shown the variation of pressure with energy density in fig. (2), which is equation of state of strange quark matter. The curve (a) is for low magnetic field and curve (b) is for high magnetic field. Whereas, curve (c) is for high magnetic field without the presence of magnetic field in s-quark. The curves (b) and (c) show little oscillating behaviour because of high magnetic field and curve (a) is smooth due to low magnetic field.

From the studies of quark matter [10], it is predicted that the mass (M) of quark star $\simeq M\odot$ ($M\odot$ solar mass) and radius (R) $\simeq 10$ km. These so-called quark stars have rather different mass - radius relationship than neutron star but for stars of mass $\simeq 1.4M\odot$, the structure parameters of quark stars are very similar to those of neutron stars.

The mass and radius for nonrotating strange quark stars are obtained by integrating the structure equations of a relativistic spherical static star composed of a perfect fluid which is derived from Einstein equation. These equations are given in ref. [11]. For a given equation of state, and given central density, the structure equations are integrated numerically with the boundary conditions $m(r = 0) = 0$, to give R and M. Though, the equation of states have little oscillating behaviour, this fact does not effect to characteristic structures of strange stars. The radius R is defined by the point where $p \simeq 0$. The total gravitational mass M, moment of inertia I, surface red shift z and the period P_0 corresponding to fundamental frequency Ω_0 are then given by $M = m(R),\ I = I(R),\ z = (1 - 2GM/Rc^2)^{-1/2} - 1$ and $P_0 = \frac{2\pi}{\Omega_0}$ respectively, where $\Omega_0 = \frac{3GM}{4R^3}$ [12]. These are presented in Table 1. Figure 3 shows the variation of mass with central density for three equation of states as illustrated in Fig. 2. We noticed from this figure that with increase in magnetic field strength, the star becomes more compact. The mass and radius decrease from 2.26 to 1.86 solar mass and from 11 km to 10 km. Similarly, the values for surface red shift, moment of inertia and fundamental period decrease, but the central density increases with magnetic field as we tabulated in table.

In conclusion, we conclude that the presence of strong magnetic field in strange quark

matter reduces the mass and radius of strange star. That is the star becomes more compact. Also, the strangeness fraction increases on the average, though, there is little oscillation due to Landau levels are passing the Fermi level.

REFERENCES

[1] M. A. Ruderman, Ann. Rev. Astron. Astrophys. **10**, 427 (1972).

[2] P. Ghosh and F. K. Lamb, Ap. J. **223** L83 (1978).

[3] J. Trumper et al., Ap. J. **219** L105 (1978); W. A. Wheaton et al., Nature **272**, 240 (1979); D. E. Gruber, et al., Ap. J. **240**, L127 (1980); T. Mihara, et al., Nature **346**, 250 (1990).

[4] R. N. Manchester and J. H. Taylor, Astron. J. **86**, 1953 (1981).

[5] R. C. Duncan and C. Thompson, Ap. J. **392** L9 (1992); C. Thompson and R. C. Duncan, Ap. J. **408**, 194 (1993); G. S. Bisnovatyi- Kogan, Astron. Astrophys. Transaction **3**, 287 (1993).

[6] E. Witten, Phys. Rev. **D30**, 272 (1984).

[7] E. Farhi and R. L. Jaffe, Phys. Rev. **D30**, 2379 (1984).

[8] L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Pergamon Press, Oxford, (1965).

[9] S. Chakrabarty, Phys. Rev. **D51**, 4591 (1995) and references therein.

[10] C. Alcock, et al., Ap. J. **310**, 261 (1986); P. Haensel, et al., Astr., Astrophys. **160**, 121 (1986).

[11] B. Datta, et al., Phys. Lett. **B283**, 313 (1992); P. K. Sahu, R. Basu and B. Datta, Ap. J. **416**, 267 (1993); S. K. Ghosh and P. K. Sahu, Int. Jour. Mod. Phys. **E2**, 575 (1993); P. K. Sahu, Study of the properties of dense nuclear matter and application to some astrophysical systems, hep-ph/9504367.

[12] C. Cutler, L. Lindblom and R. J. Splinter, Ap. J.**363**, 603 (1990).

TABLES

TABLE I. The radius (R), mass (M), surface red shift (z), moment of inertia (I) and period of fundamental frequency (P_0) of strange stars versus central density ρ_c for three cases; (a): $H = 10^{14}$ G, (b): $H = 10^{18}$ G and (c): $H = 10^{18}$ G but absence of magnetic field in s-quark.

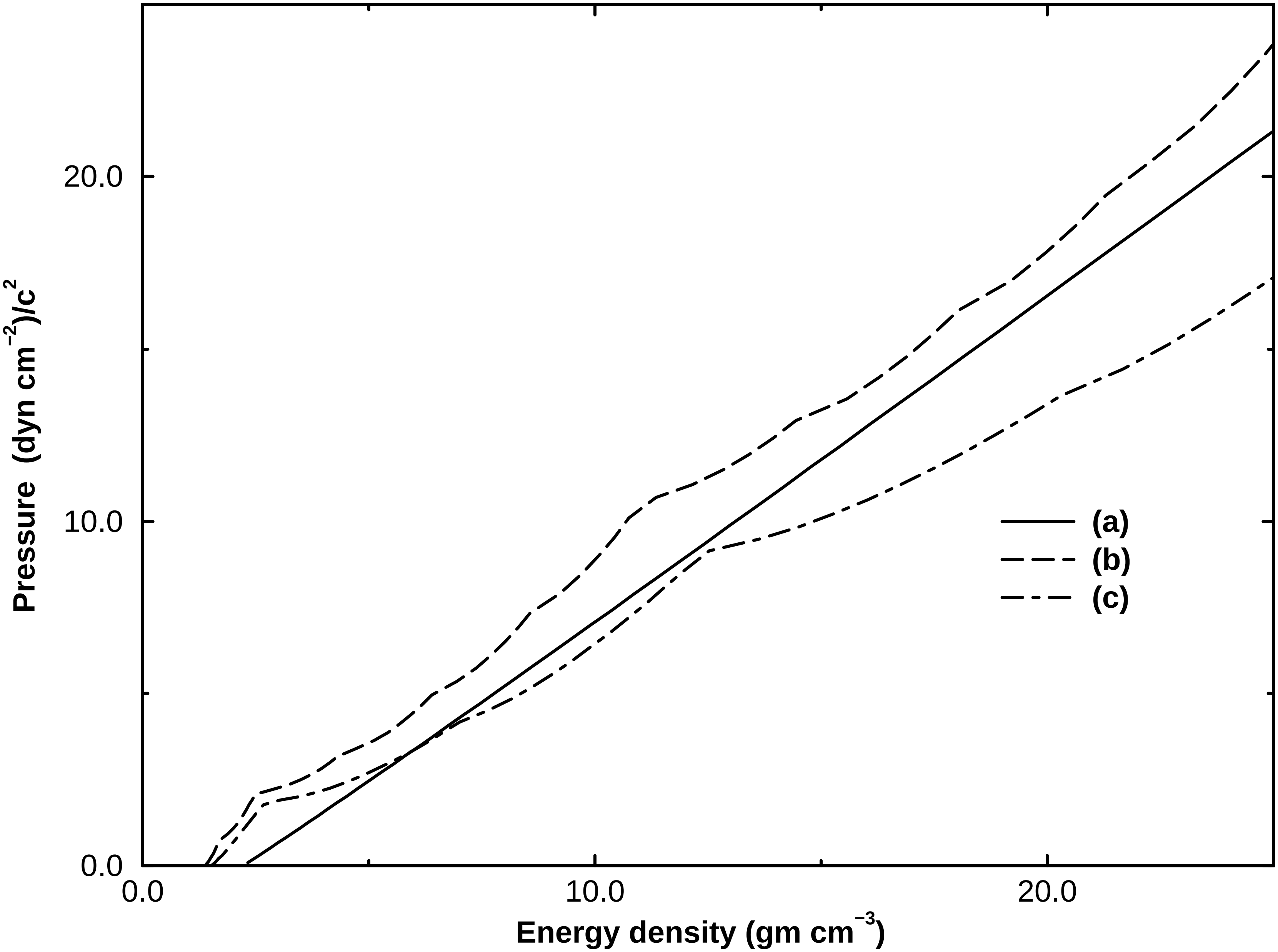
ρ_c	R	$M/M_{\odot}$	z	I	P_0	$Cases$
$(g\ cm^{-3})$	(km)			$(g\ cm^{-2})$	(ms)	
2.5×10^{15}	11.22	2.26	0.57	3.50×10^{45}	0.49	(a)
3.5×10^{15}	10.01	1.86	0.49	2.41×10^{45}	0.46	(b)
3.5×10^{15}	9.85	1.83	0.48	2.13×10^{45}	0.45	(c)

FIGURES

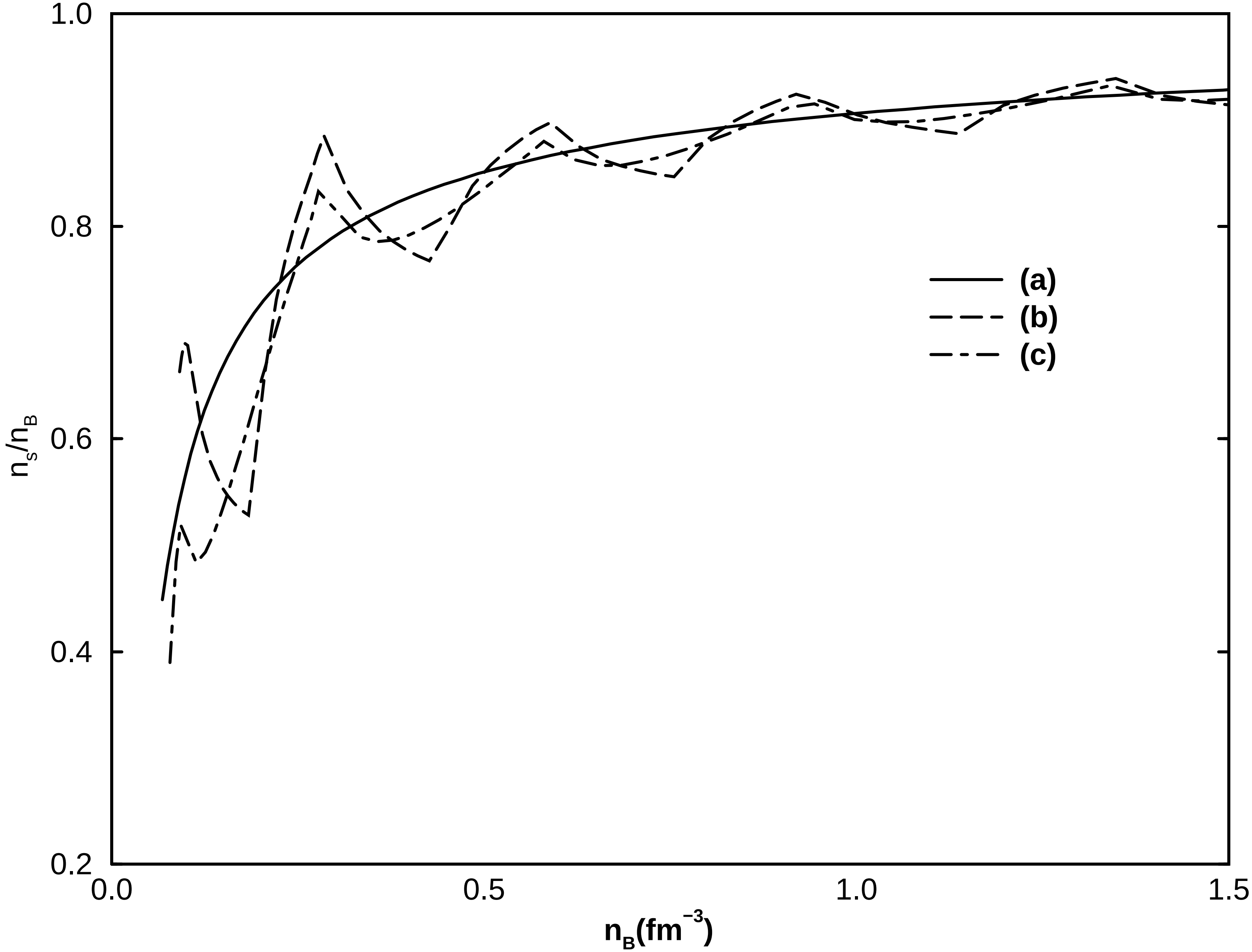
FIG. 1. Strangeness fraction and baryon density curves for three cases; Case I: $H = 10^{14}$ G (curve a); Case II: $H = 10^{18}$ G (curve b) and Case III: $H = 10^{18}$ G (curve c) but absence of magnetic field in s-quark.

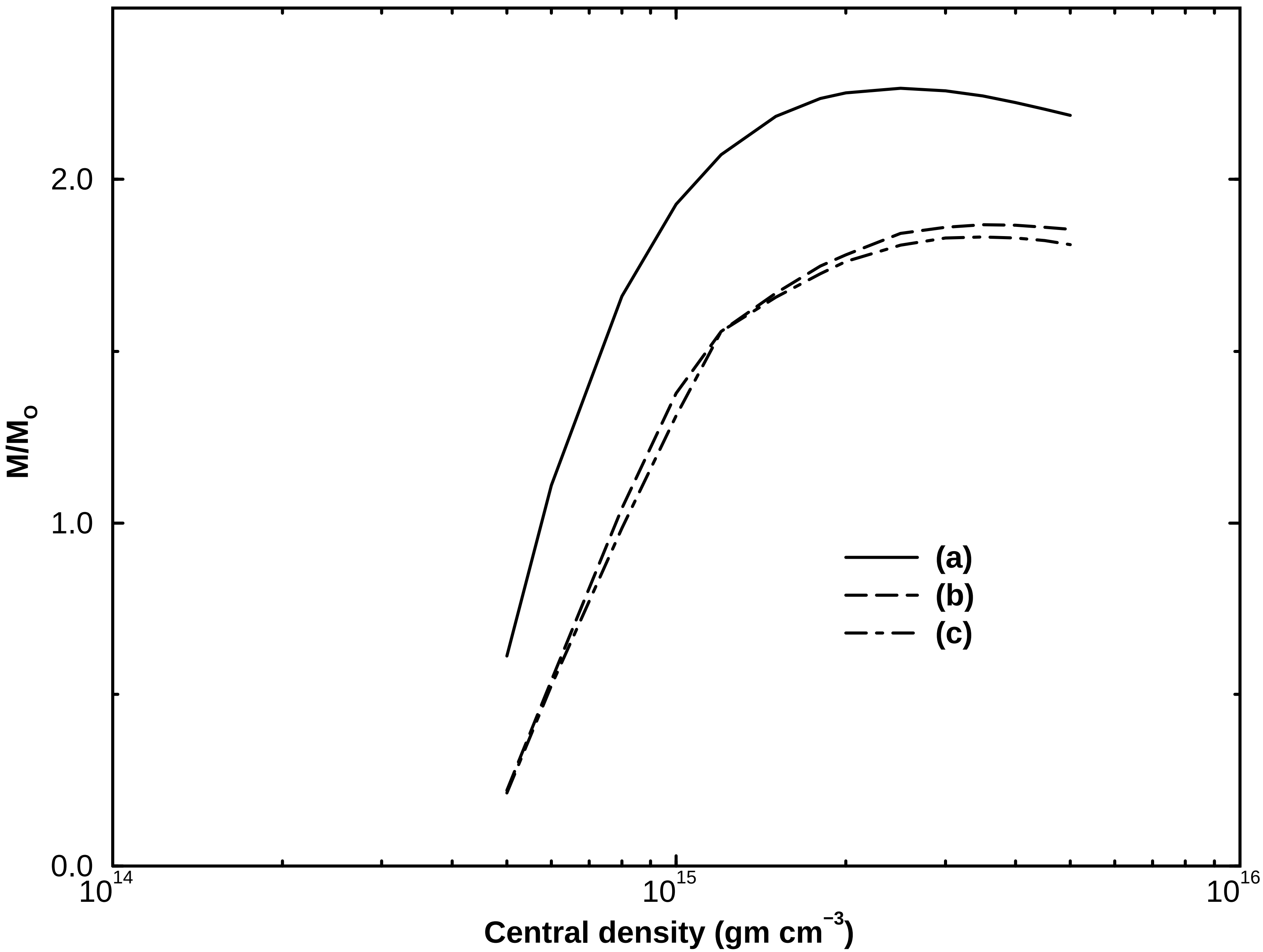
FIG. 2. Pressure and energy density curves for three cases as mentioned in figure 1.

FIG. 3. Mass and central density curves for three cases as mentioned in figure 1.



Pressure (dyn cm^{-2})/c^2
Energy density (gm cm^{-3})
0.0
10.0
20.0
0.0
10.0
20.0
(a)
(b)
(c)


1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
n_s/n_B
$n_B(fm^{-3})$
(a)
(b)
(c)



M/M$_O$
0.0
1.0
2.0
10^{14}
10^{15}
10^{16}
Central density (gm cm^{-3})
(a)
(b)
(c)